JAN 12 ARSEY

1. A geometric series has first term $a = 360$ and common ratio $r = \frac{7}{8}$

Giving your answers to 3 significant figures where appropriate, find

(a) the 20th term of the series, **(2)**

(b) the sum of the first 20 terms of the series, **(2)**

(c) the sum to infinity of the series. **(2)**

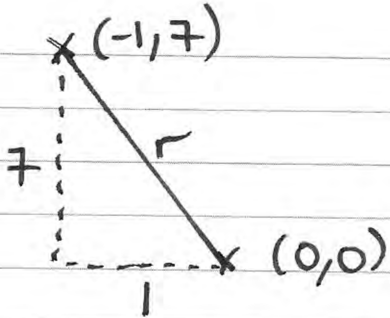
a) $U_{20} = ar^{19} = 360\left(\frac{7}{8}\right)^{19} \approx 28.5$ (3sf)

b) $S_{20} = \frac{a(1-r^{20})}{1-r} = \frac{360\left(1-\left(\frac{7}{8}\right)^{20}\right)}{\frac{1}{8}} \approx 2680$ (3sf)

c) $S_\infty = \frac{a}{1-r} = \frac{360}{\frac{1}{8}} = \underline{2880}$

2. A circle C has centre $(-1, 7)$ and passes through the point $(0, 0)$. Find

(4)



$r^2 = 1^2 + 7^2 = 50$

$(x+1)^2 + (y-7)^2 = 50$

3. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x, of

$$\left(1+\frac{x}{4}\right)^8$$

giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of $(1.025)^8$, giving your answer to 4 decimal places.

(3)

(a) $\left(1+\frac{x}{4}\right)^8$

$(a+b)^8 = a^8 + 8a^7b + \binom{8}{2}a^6b^2 + \binom{8}{3}a^5b^3$

$\binom{8}{2} = 28 \qquad \binom{8}{3} = 56$

$\therefore \left(1+\frac{x}{4}\right)^8 = 1 + 8\left(\frac{x}{4}\right) + 28\left(\frac{x}{4}\right)^2 + 56\left(\frac{x}{4}\right)^3$

$= 1 + 2x + \frac{7}{4}x^2 + \frac{7}{8}x^3$

b) $\left(1+\frac{x}{4}\right)^8 \simeq (1.025)^8 \Rightarrow \frac{x}{4} = 0.025 \Rightarrow x = 0.1$

$\therefore 1.025^8 \simeq 1 + 2(0.1) + \frac{7}{4}(0.1)^2 + \frac{7}{8}(0.1)^3$

$\simeq 1.2184$ (4dp)

4. Given that $y = 3x^2$,

(a) show that $\log_3 y = 1 + 2\log_3 x$

(b) Hence, or otherwise, solve the equation

$$1 + 2\log_3 x = \log_3 (28x - 9)$$

(3)

a) $\log_3 y = \log_3 3x^2 = \log_3 3 + \log_3 x^2$

$= 1 + 2\log_3 x$ #

b) $\log_3 y = \log_3 (28x - 9)$

$\Rightarrow y = 28x - 9 \qquad y = 3x^2$

$3x^2 = 28x - 9 \Rightarrow 3x^2 - 28x + 9 = 0$

$(3x - 1)(x - 9) = 0$

$\underline{x = \frac{1}{3}} \quad \underline{x = 9}$

5. $f(x) = x^3 + ax^2 + bx + 3$, where a and b are constants

Given that when $f(x)$ is divided by $(x+2)$ the remainder is 7,

(a) show that $2a - b = 6$

(2)

Given also that when $f(x)$ is divided by $(x-1)$ the remainder is 4,

(b) find the value of a and the value of b.

(4)

a) $f(-2) = 7$ $\quad f(-2) = -8 + 4a - 2b + 3 = 7$

$4a - 2b = 12$

$2a - b = 6$ #

b) $f(1) = 4$

$f(1) = 1 + a + b + 3 = 4 \Rightarrow a + b = 0$

$2a - b = 6$ +

$3a = 6 \Rightarrow a = 2$

$\Rightarrow \underline{b = -2}$

6.

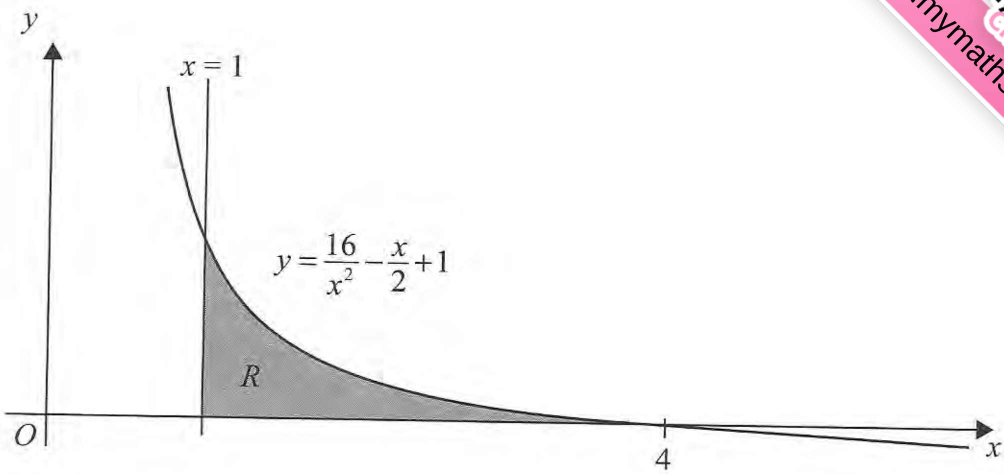


Figure 1

Figure 1 shows the graph of the curve with equation

$$y = \frac{16}{x^2} - \frac{x}{2} + 1, \qquad x > 0$$

The finite region R, bounded by the lines $x = 1$, the x-axis and the curve, is shown shaded in Figure 1. The curve crosses the x-axis at the point $(4, 0)$.

(a) Complete the table with the values of y corresponding to $x = 2$ and 2.5

$h = 0.5$

x	1	1.5	2	2.5	3	3.5	4
y	16.5	7.361	4	2.31	1.278	0.556	0

(2)

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

(c) Use integration to find the exact value for the area of R.

(5)

b) $\frac{1}{2}(0.5)\left(16.5+0+2(7.361+4+\ldots)\right) \simeq 11.88$

c) Area $= \int_1^4 \frac{16}{x^2} - \frac{x}{2} + 1\,dx = \int_1^4 16x^{-2} - \frac{1}{2}x^1 + 1\,dx$

$= \left[-16x^{-1} - \frac{1}{4}x^2 + x\right]_1^4 = \left(-\frac{16}{4} - \frac{1}{4}(4)^2 + 4\right) - \left(-16 - \frac{1}{4} + 1\right)$

$= (-4) - \left(-\frac{61}{4}\right) = \frac{45}{4}$

7.

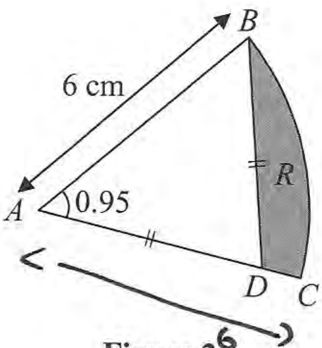


Figure 2

Figure 2 shows ABC, a sector of a circle of radius 6 cm with centre A. Given that the size of angle BAC is 0.95 radians, find

(a) the length of the arc BC, $r\theta = 6\times0.95 = \underline{5.7}$

(2)

(b) the area of the sector ABC. $\frac{1}{2}r^2\theta = \underline{17.1}$

(2)

The point D lies on the line AC and is such that $AD = BD$. The region R, shown shaded in Figure 2, is bounded by the lines CD, DB and the arc BC.

(c) Show that the length of AD is 5.16 cm to 3 significant figures.

(2)

Find

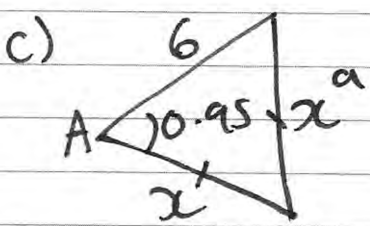
(d) the perimeter of R,

(2)

(e) the area of R, giving your answer to 2 significant figures.

(4)

c)



let $AD = x$

$a^2 = b^2 + c^2 - 2bc\cos A$

$\cancel{x^2} = \cancel{x^2} + 6^2 - 2(x)(6)\cos 0.95$

$12x\cos 0.95 = 36$

$\Rightarrow x = \dfrac{36}{12\cos 0.95} \approx 5.16\ (3sf)$

#

Question 7 continued

$DC = 6 - x \approx 0.843 (3sf)$

$\text{Perimeter} = x + (6-x) + 5.7 \quad = \underline{11.}$

d)

$\text{area } \triangle ABD = \frac{1}{2} ab\sin C$

$= \frac{1}{2}(6)(5.16...) \sin 0.95$

≈ 12.6

$\therefore \text{Area } R = 17.1 - 12.6 \quad \approx \underline{4.5} (2sf)$

8.

arc $= \frac{2\pi x}{4} = \frac{1}{2}\pi x$

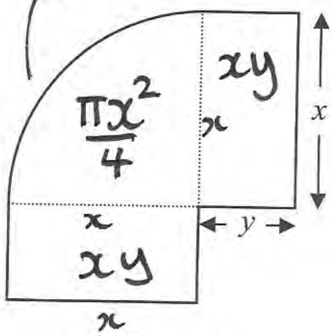


Figure 3

Figure 3 shows a flowerbed. Its shape is a quarter of a circle of radius x metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to x metres and width equal to y metres.

Given that the area of the flowerbed is 4 m^2,

$\frac{\pi x^2}{4} + 2xy = 4$

(a) show that

$$y = \frac{16 - \pi x^2}{8x}$$

$2xy = \cancel{4}\frac{16}{4} - \frac{\pi x^2}{4}$

$\therefore \quad y = \frac{16 - \pi x^2}{8x}$ #

(3)

(b) Hence show that the perimeter P metres of the flowerbed is given by the equation

$$P = \frac{8}{x} + 2x$$

$P = \frac{1}{2}\pi x + y + x + y + y + x + y$

$P = \frac{1}{2}\pi x + 2x + 4y$

(3)

(c) Use calculus to find the minimum value of P.

(5)

(d) Find the width of each rectangle when the perimeter is a minimum.
Give your answer to the nearest centimetre.

(2)

b) $P = \frac{1}{2}\pi x + 2x + 4\left(\frac{16 - \pi x^2}{8x}\right)$

$P = \cancel{\frac{1}{2}\pi x} + 2x + \frac{8}{x} - \cancel{\frac{1}{2}\pi x} = \frac{8}{x} + 2x$ #

c) $P = 8x^{-1} + 2x$

$\frac{dP}{dx} = -8x^{-2} + 2 = \frac{-8}{x^2} + 2$

Min Value $\frac{dP}{dx} = 0$

$\frac{8}{x^2} = 2 \Rightarrow x^2 = 4$

$\Rightarrow x = 2$

$$\min P = \frac{8}{2} + 2(2) = 8$$

d) Width $= y$ $\qquad y = \frac{16 - \pi(2)^2}{8(2)} = 1 - \frac{4\pi}{16}$

$\therefore \; y = 1 - \frac{1}{4}\pi = 0.2146\ldots \text{ m}$

$\simeq 21 \text{ cm}$

9. (i) Find the solutions of the equation $\sin(3x - 15°) = \frac{1}{2}$, for which 0 ... $0°$

$0 \leq 3x$

(ii)

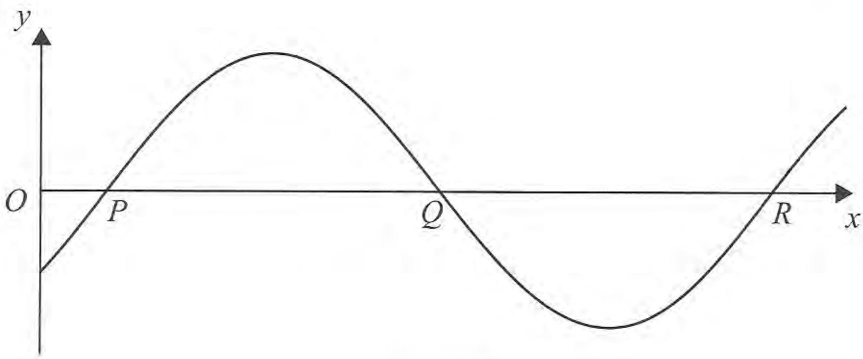


Figure 4

Figure 4 shows part of the curve with equation

$$y = \sin(ax - b), \text{ where } a > 0,\ 0 < b < \pi$$

The curve cuts the x-axis at the points P, Q and R as shown.

Given that the coordinates of P, Q and R are $\left(\frac{\pi}{10}, 0\right)$, $\left(\frac{3\pi}{5}, 0\right)$ and $\left(\frac{11\pi}{10}, 0\right)$ respectively, find the values of a and b.

(4)

$3x - 15 = \sin^{-1}\left(\frac{1}{2}\right) = 30°, 150°, 390°, 510°,$

$\therefore\ 3x = 45, 165, 405, 525$

$x = \underline{15}, \underline{55}, \underline{135}, \underline{175}$

b) $\sin(ax - b) = 0 \Rightarrow ax - b = \sin^{-1}(0) = 0, \pi, 2\pi$

$+b$ $\quad ax = b, \pi + b, 2\pi + b$ $\quad \div a$ $\quad x = \frac{b}{a}, \frac{\pi + b}{a}$

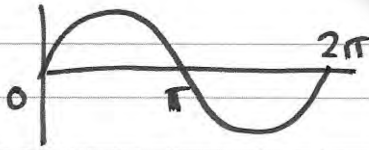
$= \frac{\pi}{10} \quad = \frac{3\pi}{5}$

$\therefore \frac{b}{a} = \frac{\pi}{10} \qquad \frac{\pi}{a} + \frac{b}{a} = \frac{3\pi}{5}$

$\Rightarrow \frac{b}{a} = \frac{\pi}{10} \Rightarrow \frac{\pi}{a} = \frac{3\pi}{5}\overset{\times 2}{\underset{\times 2}{}} - \frac{\pi}{10} \qquad \frac{\pi}{a} = \frac{5\pi}{10} = \frac{\pi}{2}$

$\therefore a = 2 \quad b = \frac{\pi}{5}$

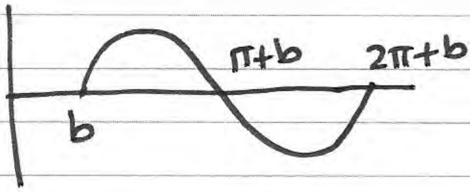
alt2 $f(x) = \sin x$



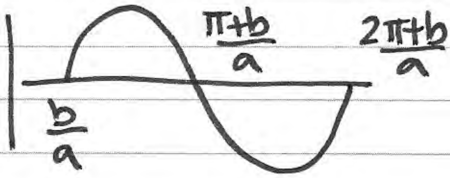
$f(ax-b) = \sin(ax-b)$

$\overrightarrow{b}$ then $\rightarrow a \leftarrow$

$\overrightarrow{b}$



$\rightarrow a \leftarrow$



$\therefore \frac{b}{a} = \frac{\pi}{a}$ $\quad \frac{\pi+b}{a} = \frac{3\pi}{5}$ $\quad$ repeat (b)

alt3

$(\frac{\pi}{10}, 0) \Rightarrow \sin(\frac{\pi}{10}a - b) = 0 \Rightarrow \frac{\pi}{10}a - b = 0$ ①

$(\frac{3\pi}{5}, 0) \Rightarrow \sin(\frac{3\pi}{5}a - b) = 0 \Rightarrow \frac{3\pi}{5}a - b = \pi$ ②

② − ① $\Rightarrow \frac{3\cancel{\pi}}{5}a - \frac{\cancel{\pi}}{10}a = \cancel{\pi}$

$\frac{6}{10}\cancel{\frac{3}{5}}a - \frac{1}{10}a = 1$

$\frac{5}{10}a = 1$

$a = \frac{10}{5}$

① $\frac{\pi}{10}(2) - b = 0 \Rightarrow b = \frac{2\pi}{10} = \frac{1}{5}\pi$

$a = 2$

$a = 2 \quad b = \frac{1}{5}\pi$